Search Models of the Labor Market

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UC3M

Macroeconomics III

Types of Labor Market Risk

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- So far, we took earnings uncertainty as exogenous.
- Unemployment is a major risk workers face. Finding a job takes time.
- Not all jobs are the same, which creates risk.
- My job may develop well, or poorly.
- I may, or may not, find a better job while employed.

Heterogeneous Jobs

Two famous studies suggesting that firms matter for individual wages:

- Abowd et al. (1999) show that wages are different across firms.
- Topel and Ward (1992) finds that job-to-job transition contribute significantly to individual's life-cycle wage growth.

Abowd et al. (1999): The Approach

- How important are worker and firm effects for wages.
- French matched employer-employee panel data.
- Estimate: $log(y_{i,j,t}) = \beta X_{i,t} + \theta D + \phi F + \epsilon_{i,j,t}$.
- θ gives individual and ϕ firm fixed effect.
- With enough mobility, we can estimate it.

Abowd et al. (1999): Key Assumptions

- No selection on ϵ allowed.
- Log wages are linear in firm and worker component.

Abowd et al. (1999): Industry Differential

Independent Variable	Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error
Based on Order-Independent Estimates						
Industry Average α	1.0390	(0.0023)	1.0053	(0.0022)		
Industry Average ψ	-0.0220	(0.0006)			0.0683	(0.0005)
Intercept	3.3023	(0.0019)	3.3031	(0.0019)	3.0935	(0.0018)
R^2	0.8487		0.8425		0.0682	
Based on Order-Dependent Estimates: Persons First						
Industry Average α	0.8011	(0.0019)	0.8324	(0.0017)		
Industry Average ψ	0.2410	(0.0151)			-0.6659	(0.0150)
Intercept	3.1126	(0.0019)	3.1088	(0.0018)	3.0687	(0.0019)
R^2	0.9580		0.9213		0.2486	

- Relate industry premium to industry average person and firm effects.
- Firm effects: Between 7 and 25%.
- In general, worker effects more important.

What They Do?

- Topel and Ward (1992) follow workers for the first ten years of their careers in the LEED 57-72.
- They study employment stability during these year.
- They find that wages grow by 66%.
- Where does this incredible growth come from?
- Wage gains on the job.
- Wage gains at job changes.

Employment Stability

TABLE I
AGE DISTRIBUTIONS AT ONSET OF CONTINUOUS WORK—SIXTEEN YEARS OF AGE AND OLDER ($N = 8{,}102$)

	Age at beginning of employment spell							
Length of spell	≤ 18	19	20	21	22	23	24	≥ 25
≥ 1 quarter	46.0	25.6	14.7	7.6	4.1	1.5	0.5	0.1
≥ 2 quarters	29.6	24.9	18.8	11.4	8.1	4.8	1.7	0.7
≥ 3 quarters	24.2	22.7	18.6	12.9	10.1	7.0	2.8	1.5
≥ 1 year	21.6	21.0	17.3	13.6	11.8	8.3	3.9	2.6

- Entry up to age 20.
- Early: Unstable jobs.

Employment Mobility

	Potential market experience (years)									
	1	2	3	4	5	6	7	8	9	10
Actual market experience	0.70	1.36	2.10	2.89	3.73	4.61	5.49	6.38	7.29	8.19
Additional experience	0.70	0.66	0.74	0.79	0.84	0.88	0.88	0.89	0.91	0.90

- Substantial time in non-employment early in life.

	Cumulative full-time jobs										
	1	2	3	4	5	6	7	8	9	10	11–15
Percent with indicated number of jobs	4.3	7.0	9.9	11.1	11.6	9.5	9.0	8.2	6.9	4.8	13.0

- Large number of jobs early in life.

Exit Hazard

- Hazard decreases in tenure.
- Not driven by experience.

Job-to-Job Transitions

		Current job tenure (quarters)									
		1	2	3	4	5	6	7	8	9	10
Prior experience:											
None	*j-j*	0.093	0.078	0.059	0.070	0.068	0.066	0.054	0.072	0.065	0.050
	j-n	0.297	0.118	0.091	0.080	0.099	0.059	0.050	0.047	0.054	0.035
1 year or less	*j-j*	0.150	0.122	0.094	0.083	0.083	0.064	0.078	0.075	0.068	0.076
	j-n	0.262	0.125	0.073	0.062	0.075	0.050	0.036	0.039	0.045	0.034
1–2 years	*j-j*	0.183	0.144	0.109	0.089	0.084	0.079	0.083	0.078	0.067	0.063
	j-n	0.156	0.094	0.060	0.040	0.044	0.048	0.025	0.031	0.026	0.023
2–4 years	*j-j*	0.186	0.151	0.120	0.096	0.086	0.078	0.074	0.070	0.067	0.060
	j-n	0.109	0.075	0.043	0.036	0.036	0.029	0.025	0.027	0.025	0.021
4–7 years	*j-j*	0.193	0.145	0.114	0.098	0.080	0.067	0.071	0.069	0.054	0.060
	j-n	0.078	0.057	0.035	0.029	0.121	0.029	0.020	0.017	0.022	0.018
More than 8 years	*j-j*	0.175	0.140	0.109	0.092	0.070	0.075	0.065	0.049	0.042	0.044
	j-n	0.048	0.049	0.032	0.025	0.026	0.019	0.011	0.014	0.008	0.008

- JN transitions fall in tenure.
- JTJ transitions fall in tenure.
- Supports job shopping.

Wage Growth

- Why do wages grow this much in the first 10 years?
- On the job wage growth.
- Between jobs wage growth.

$$ln(w_{i,j,t}) = \alpha_i + \beta X_{i,t} + \phi_j + \epsilon_{i,t}$$

Wages depend on firm effects ϕ_j.

Within Job Wage Growth

- $\Delta ln(w_{i,t}) = \Delta H(X_{i,t}, T_{i,t}) + \Delta\epsilon_{i,t}$
- First difference eliminates firm fixed effects.
- Concave in tenure and experience.
- More durable jobs lead more wage growth.
- The data suggests a random walk with drift and a transitory component.

Between Job Wage Growth

A. Average wage changes at job transitions as a component of wage growth: experience interval (years)

	0–2.5	2.5–5	5–7.5	7.5–10	0–10
Average wage change at job transitions	0.171	0.119	0.079	0.057	0.114
	(0.015)	(0.016)	(0.015)	(0.016)	(0.007)
Average wage gain at job transitions	0.145	0.099	0.064	0.046	0.094
	(0.015)	(0.016)	(0.015)	(0.016)	(0.007)

- On average, 10 percent extra wage growth.
- Declines in experience.
- Larger when moving to more durable jobs.
- 1/3 of total wage growth over the first 19 years due to JTJ.

A Baseline Search Model

On-the-Job Search Model

- We now formalize these ideas in a formal model.
- For the moment, we have only unemployment risk and the risk of heterogeneous jobs.
- We will consider risk on the job later.
- Search frictions are the underlying source of these risks.
- the model is in partial equilibrium. We do not model the process of job creation and wage formation.

The environment

- Workers are infinitely lived and discount future with β.
- Utility is linear in income.
- When unemployment, receive benefits b and receive a job offer with probability λ that they can accept or reject.
- When employed, receive a wage w and receive a job offer with probability λ_e that they can accept or reject. They lose their current job and become unemployed with probability δ.
- Job offers are random draws from a continuous distribution with CDF $F(w)$.

Value Function Unemployed

The value of being unemployed:

$$V^U = b + \beta \left[(1-\lambda) V^U + \lambda \int_{\underline{w}}^{\overline{w}} \max \left\{ V^E(w'), V^U \right\} dF(w') \right].$$

The unemployed accept any job better than w^*:

$$V^U = b + \beta \left[(1-\lambda) V^U + \lambda \left(F(w^*) V^U + \int_{w^*}^{\overline{w}} V^E(w') dF(w') \right) \right].$$

Value Function Employed

The value of being employed:

$$V^E(w) = w + \beta\Big[\delta V^U + (1-\delta)\Big[(1-\lambda_e)\max\{V^E(w), V^U\} + \lambda_e \int_{\underline{w}}^{\overline{w}} \max\{V^E(w), V^E(w'), V^U\} dF(w')\Big]\Big].$$

The employed accept any job better than w:

$$V^E(w) = w + \beta\Big[\delta V^U + (1-\delta)\Big[(1-\lambda_e) max\{V^E(w), V^U\} + \lambda_e\Big(F(w)\max\{V^E(w), V^U\} + \int_{w}^{\overline{w}} \max\{V^E(w'), V^U\} dF(w')\Big)\Big]\Big].$$

Stationary Equilibrium

A stationary equilibrium is characterized by

Stationary Equilibrium

A stationary equilibrium is characterized by

- Policy functions for the unemployed w^*, employed $(\psi^E(w))$, and outside offers $(\psi^J(w, w'))$.
- A stationary distribution of workers over employment and job states $\Psi(E, w)$.

Analytical Characterization

Stationary Equilibrium

- It turns out, we can characterize the solution to a large class of search models when time is continuous.
- Continuous time helps to characterize optimal policy w^*.
- To gain intuition, we will now study the problem in continuous time.
- Afterward, we will go back to the discrete time case for a more complicated model and numerical solutions.

The Problem in Continuous Time

Asset values of employment and unemployment:

$$
\begin{aligned}
rW(w) &= w + \lambda_e \int_{w}^{w_{max}} [W(z) - W(w)]dF(z) \\
&\quad - \delta(W(w) - U) \\
rU &= b + \lambda \int_{w^*}^{w_{max}} [W(z) - U]dF(z).
\end{aligned}
$$

Evaluate at reservation wage policy $W(w^*) = U$:

$$
\begin{aligned}
rW(w^*) &= w^* + \lambda_e \int_{w^*}^{w_{max}} [W(z) - W(w^*)]dF(z) - \delta(W(w^*) - U) \\
&= w^* + \lambda_e \int_{w^*}^{w_{max}} [W(z) - W(w^*)]dF(z) \\
&= b + \lambda \int_{w^*}^{w_{max}} [W(z) - W(w^*)]dF(z)
\end{aligned}
$$

Optimal Policy

$$w^* = b + (\lambda - \lambda_e) \int_{w^*}^{w_{max}} [W(z) - W(w^*)] dF(z)$$

- Reservation wage is outside value plus value of search.
- Use integration by parts to get:

$$w^* = b + (\lambda - \lambda_e) \int_{w^*}^{w_{max}} [W'(z)[1 - F(z)] dz]$$

Optimal Policy II

Use value of employment with Leibnitz integral rule to derive:

$$W'(z) = \frac{1}{r + \delta + \lambda_e[1 - F(z)]}$$

$$w^* = b + (\lambda - \lambda_e) \int_{w^*}^{w_{max}} \left[\frac{1 - F(z)}{r + \delta + \lambda_e[1 - F(z)]} dz \right]$$

- Characterizes implicitly w^*.
- High r and δ decrease value of search.

Stationary Distribution

- Let $G(w)$ be the CDF of employed workers over w, i.e., the mass of workers with wage at most w.
- In a stationary equilibrium, the inflow to $G(w)$ needs to match its outflow.

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- In a stationary equilibrium, the inflow to $G(w)$ needs to match its outflow.

$$\underbrace{u\lambda(F(w) - F(w^*))}_{Inflow} = \underbrace{(1-u)G(w)[\delta + \lambda_e(1 - F(w))]}_{Outflow}$$

Stationary Distribution II

Evaluating at $G(w^{max}) = 1$ gives an implicit solution for u:

$$\frac{1-u}{u} = \frac{\lambda(1-F(w^*))}{\delta}$$

Solving for $G(w)$:

$$G(w) = \frac{F(w)-F(w^*)}{1-F(w^*)} \frac{\delta}{\delta + \lambda_e[1-F(w)]}.$$

- High job destruction allocates workers to the left of distribution.
- High on the job search efficiency, allocates workers to the right.

Endogenous Job Search

- So far, job offer arrival rates are exogenous.
- Search incentives are not the same for everyone.
- Many report not searching in employment or non-employment.
- We may want to endogenize the search decision.

Endogenous Job Search

$$V^E(w) = \max_s \Big\{ w - c(s) + \beta \Big[\delta V^U + (1-\delta)\Big((1-s)max\{V^E(w), V^U\} + s\int_{\underline{w}}^{\overline{w}} max\{V^E(w), V^E(w'), V^U\} dF(w')\Big)\Big]\Big\}$$

$$V^U = \max_s \Big\{ b - c(s) + \beta \Big[(1-s)V^U + s\int_{\underline{w}}^{\overline{w}} max\Big\{V^E(w'), V^U\Big\} dF(w')\Big]\Big\}$$

- Workers generate an offer with probability s at cost $c(s) = \eta_0 \frac{s^{\eta_1+1}}{\eta_1+1}$.
- Incentives largest for the unemployed and poorly matched.

Search Frictions and the Law of One Price

Law of One Price

- About 2/3 of wage inequality unexplained by observables.
- Moving between jobs implies wage dynamics.
- Job-to-job transitions important for wage growth.
- Importance of the job component for inequality?
- $log(y_{i,j,t}) = \beta X_{i,t} + \nu_i + \phi_j + \epsilon_{i,j,t}$.

What We Need to Know

Estimating the contribution of "luck": ϕ_j.

- Either measure $Var(\phi_j)$ in the data.

  The main problem is differentiating ϕ_j from ν_i.

- Infer wage offer distribution from the data/model.

What We Need to Know

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- Infer wage offer distribution from the data/model.

  Model the selection from offers to accepted matches.

- Measure something related.

Hornstein et al. (2012)

Idea

- Knowing $G(w) = \frac{F(w)-F(w^*)}{1-F(w^*)} \frac{\delta}{\delta+\lambda_e[1-F(w)]}$ is hard.
- Particularly wage offer distribution, $F(w)$, is difficult to infer.
- Good information on worker flow rates available.
- New measure of wage dispersion which only depends on flows.

A Simple Search Model

- Start with model with search in unemployment and permanent wage differences.
- Risk neutral workers, discount at rate r.
- Unemployed receive: $b = \rho\overline{w}$.
- Sample offers with probability λ_u from distribution $F(w)$.
- Matches destroyed with probability σ.

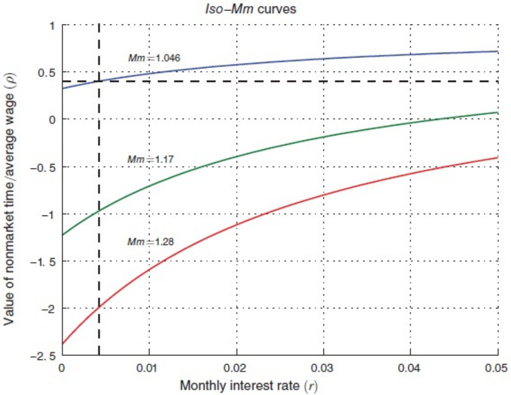
A Simple Search Model

The mean-min ratio is independent of the wage offer distribution.

$$Mm = \frac{\frac{\lambda_u}{r+\sigma} + 1}{\frac{\lambda_u}{r+\sigma} + \rho}$$

- High λ_u increases value of waiting.
- High ρ increases value of waiting.
- High r or σ decrease value of waiting.

Quantitative Implications



- Large wage dispersion only with negative replacement rates.

Intuition for Result

We have seen that workers follow reservation wage strategy:

$$w^* = b + \lambda \int_{w^*}^{w_{max}} \left[\frac{1 - F(z)}{r + \delta} dz\right].$$

- In the data, λ is large (0.15-0.3 monthly).
- Workers do not find it worthwhile to stay unemployed for long.
- Value of search must be low: $\left(\int_{w^*}^{w_{max}} [\frac{1-F(z)}{r+\delta} dz]\right)$.
- $F(z)$ cannot be very dispersed.

Results Robust to

- Stochastic wages.
- Returns to experience.
- Risk aversion (self-insurance).
- Directed search.

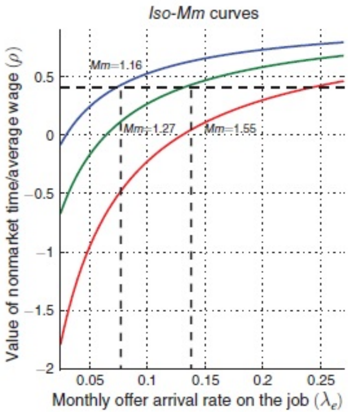
On-the-Job Search

With on-the-job search, the Mm ration becomes:

$$Mm = \frac{\frac{\lambda_u - \lambda_e}{r+\sigma+\lambda_e} + 1}{\frac{\lambda_u - \lambda_e}{r+\sigma+\lambda_e} + \rho}$$

- On-the-job search reduces option value of unemployment.
- Mm ratio increases.

Quantitative Implications



- With on-the-job search, frictional wage dispersion can become large.
- Can become huge with tenure contracts. Real world?

Relation to Literature

- Estimate model, impose flows, value of unemployment, and discounting: Find large worker heterogeneity or measurement error.
- Leave value of unemployment or discounting unrestricted: Large frictional dispersion with strange parameters.

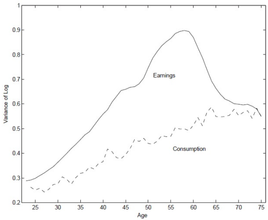
Tjaden and Wellschmied (2014)

Tjaden and Wellschmied (2014)

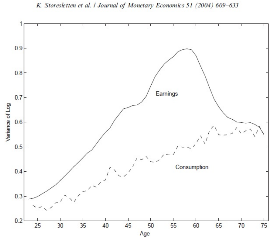
- Wage heterogeneity because of job heterogeneity and stochastic worker productivity.
- On-the-job search and learning imply large Mm.
- Build a model that has low w^*.
- Does this imply large contribution to variance of log wages?
- Dispersion of wage offer distribution limits role of search frictions.
- Identify model by second moments of wages over the life-cycle.

Identification

K. Storesletten et al. / Journal of Monetary Economics 51 (2004) 609–633



Identification



- Knowing wage offer distribution, initial dispersion identifies worker heterogeneity.
- Increase over the life-cycle identifies innovations to wages.
- Knowing wage distribution (policy), second moments of wage growth identify the offer distribution.
- Important to account for wage losses (reallocation shocks).

A Simple Model

Begin by emphasizing importance of reallocation offers for wage dispersion:

- Not all job-to-job transitions are value improving.
- Workers receive offer with λ_d which they accept or move to non-employment.

$$rW(w) = w + \lambda(1-\lambda_d)\int_{w}^{w_{max}}[W(z)-W(w)]dF(z)$$
$$+\lambda\lambda_d\int_{w^*}^{w_{max}}[W(z)-W(w)]dF(z)$$
$$-(\omega+\lambda\lambda_d F(w^*))(W(w)-U).$$
$$rU = b + \lambda_u\int_{w^*}^{w_{max}}[W(z)-U]dF(z).$$

Job Offer Arrival Rate

$$JTJ = \lambda(1-\lambda_d)\underbrace{\int_{w^*}^{w_{max}}[1-F(z)]dG(z)}_{=:ANO} + \lambda\lambda_d\underbrace{[1-F(w^*)]}_{=:ARO},$$

$$\lambda^* = \frac{JTJ}{(1-\lambda_d)ANO + \lambda_d ARO}.$$

How is $G(w)$ affected?

$$G(w) = \frac{F(w)-F(w^*)}{1-F(w^*)}\frac{\overbrace{\omega+\lambda^*\lambda_d}^{=:D}}{\underbrace{\omega+\lambda^*\lambda_d}_{=:D}+\underbrace{\lambda^*(1-\lambda_d)}_{=:C}[1-F(w)]}.$$

Effects of λ_d

Figure: Wage CDF $G(w)$

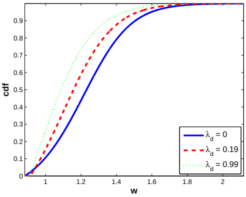
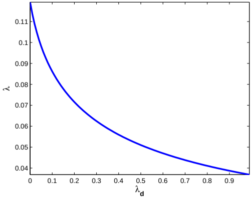


Figure: Implied λ



- CDF becomes steeper and λ falls.
- Particularly for low values of λ_d.

Non Value-Improving JTJ

Table: Wage Cuts after Job to Job Transitions

Sample		Share loss	Mean loss
Whole		0.344	-0.196
Job characteristics			
	- NU-U	0.346	-0.196
	- HI	0.352	-0.196
	- Educ	0.352	-0.196

- 1/3 of workers have wage cuts at job-to-job transition.
- Not driven by compensating differentials.
- Not driven by future wage growth.

Extended Model

- Extend model to worker heterogeneity and low w^*.
- At birth, log productivity drawn from $N \sim N(\mu_N, \sigma_N^2)$.
- Meeting a firm, log productivity drawn from $F(\Gamma)$: $w_t = exp(A_t + \Gamma)$.

$$A_{t+1} = \begin{cases} A_t + \nu + \epsilon_t & \text{if } \textit{employed} \\ A_t - \delta + \epsilon_t & \text{if } \textit{unemployed}. \end{cases}$$

- Wages are random walk with drift: $\epsilon \sim N(0, \sigma_\epsilon^2)$.
- Learning by doing.
- Skill depreciation in unemployment.

Value Functions

Employed:

$$W(A_t,\Gamma) = w_t(A_t,\Gamma) + \beta(1-\phi)\mathbb{E}_t\Big\{(1-\omega) \\ \Big[(1-\lambda)H + \lambda[(1-\lambda_d)\Omega_E + \lambda_d\Lambda]\Big] + \omega U(A_{t+1})\Big\}$$

Unemployed:

$$U(A_t) = b(A_t) + Z(A_t) + \beta(1-\phi)\mathbb{E}_t\Big\{(1-\lambda_u)U(A_{t+1}) \\ + \lambda_u \int_{\Gamma_m}^{\Gamma_M} \max\{W(A_{t+1},\Gamma), U(A_{t+1})\}dF(\Gamma)\Big\}.$$

Bringing the Model to the Data

Following Topel and Ward (1992), wages in the data follow:

$$ln(w_{i,t}) = \alpha_0 + \alpha_1 d_t + \alpha_2 \mathbf{Z}_i + \beta_2 \Gamma_i + e_{i,t}$$

$$e_{i,t} = r_{i,t} + A_{i,t}.$$

- Mobility is endogenous. Observe only $\Gamma^{obs}, \epsilon^{obs}$.
- Selection also present in the model.
- We look through the model at the data!

Identifying Distributions

Wage growth between jobs and on the job:

$$\Delta ln(w^b_{i,t}) = \nu + \kappa_t + [\Gamma^{obs}_i - \Gamma^{obs}_{i-1}] + \epsilon^{obs}_{i,t} + \Delta r_{i,t}$$

$$\Delta ln(w^w_{i,t}) = \nu + \kappa_t + \epsilon^{obs}_{i,t} + \Delta r_{i,t}$$

Excess variance of job switchers over stayers identifies wage offer distribution:

$$Var\left[\Delta ln(\hat{w}^b_{i,t})\right] - Var\left[\Delta ln(\hat{w}^w_{i,t})\right]$$
$$= Var\left[\Gamma^{obs}_i - \Gamma^{obs}_{i,-1}\right] + Cov\left[\epsilon^{obs}_{i,t}(\Gamma^{obs}_i - \Gamma^{obs}_{i,-1})\right]$$

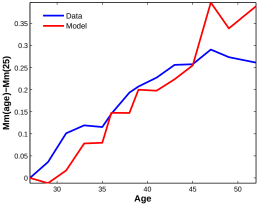
Identifying Distributions II

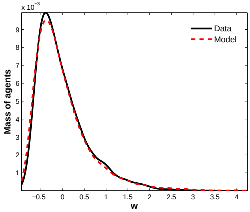
- Life-cycle profile of wage dispersion identifies σ_ϵ.
- Measurement error potentially important for quantity of wage cuts.
- Estimate $MA(12)$ process for measurement error by Kalman filter.
- $\Rightarrow$ 60% of wage losses due to reallocation shocks.

Model Fit

Table: Residual Wage Dispersion

Mean-Min Ratio				Gini		$Var(\log(\tilde{w}_{it}))$	
		Model	Data	Model	Data	Model	Data
	1^{st}	3.01	3.02				
Pctl.	5^{th}	2.21	2.14	0.24	0.29	0.18	0.21
	10^{th}	1.89	1.83				





The Importance of the Search Friction

$$Var(ln(w_i)) = Var(A_i) + Var(\Gamma_i) + 2Cov(A_i, \Gamma_i) + Var(r_i).$$

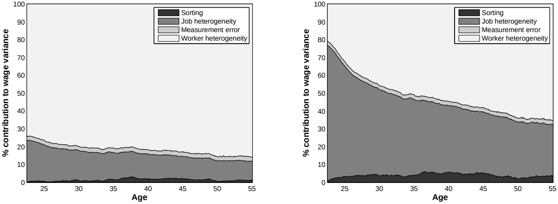


Figure: Contribution of Search Frictions to Overall Wage Dispersion Baseline v. Job Ladder Model

- On average, 13.7% of wage inequality is frictional.
- Pure job-ladder-model: 38.8%.

The Importance of Reallocation Shocks

Without reallocation shocks, workers very well sorted. Small positive wage growth at job-to-job transitions.

Table: Wage Offer Distribution and Idiosyncratic Risk

Specification	σ_F	σ_ϵ	σ_N	λ
Baseline	0.163	0.016	0.293	0.043
job ladder model $(\lambda_d = 0)$	0.296	0.017	0.117	0.1

Low et al. (2010)

Wage Process

Consider the following wage process:

$$ln(w_{i,j,t}) = d_t + x_{i,t}\beta + u_{i,t} + e_{i,t} + \phi_j$$
$$u_{i,t} = u_{i,t-1} + \varsigma_{i,t}$$
$$\Delta ln(w_{i,j,t}) = \Delta d_t + \Delta x_{i,t}\beta + \Delta e_{i,t} + \varsigma_{i,t} + M_{i,t}[\phi_j - \phi_{j-1}].$$

- Observables d_t, $x_{i,t}$.
- Transitory shocks $e_{i,t}$.
- Permanent shocks $\varsigma_{i,t}$.
- Job fixed-effects ϕ_j.

Ignoring Selection

Without selection:

$$
\begin{aligned}
g^w_{i,t} &= ln(w_{i,j,t}) - ln(w_{i,j,t}) \ \text{ if } M == 0 \\
g^b_{i,t} &= ln(w_{i,j,t}) - ln(w_{i,j,t}) \ \text{ if } M == 1 \\
Var(g^w_{i,t}) &= \sigma^2_\varsigma + 2\sigma^2_e \\
Var(g^b_{i,t}) &= \sigma^2_\varsigma + \sigma^2_\phi + 2\sigma^2_e \\
Cov(g^w_{i,t}, g^w_{i,t-1}) &= \sigma^2_e
\end{aligned}
$$

- Wage growth of stayers identify variance of permanent shocks.
- Wage growth of switchers identify variance of job effects.
- Covariances identify transitory variance.

What Type of Selection

- After bad productivity shock, go to non-employment, switch employment.
- Workers are not randomly distributed over jobs.
- Good outside offers increase mobility.
- Control for selection without structural model.

Correcting for Selection

▸ The Heckit model

Estimate participation and mobility decision:

$$P^*_{it-1} = \alpha z_{it-1} + \pi_{it-1}, \;\; P_{it-1} = 1\left\{P^*_{it-1} > 0\right\},$$
$$P^*_{it} = \alpha z_{it} + \pi_{it}, \;\; P_{it} = 1\left\{P^*_{it} > 0\right\},$$
$$M^*_{it} = \theta \kappa_{it} + \mu_{it}, \;\; M_{it} = 1\left\{M^*_{it} > 0\right\}.$$

- $z_{i,t}$ and $\kappa_{i,t}$ are worker observables.
- $(\pi_{i,t}, \pi_{i,t-1}, \mu_{i,t}) \sim N(0, I)$ and uncorrelated.

Correcting for Selection II

Observed wage growth:

$$E[\Delta w_{i,t}|P_{i,t} = 1, P_{i,t-1} = 1] = \beta\Delta x_{i,t} + G_{i,t}$$
$$g_{i,t} = \Delta w_{i,t} - \beta\Delta x_{i,t} = \underbrace{[\phi_j - \phi_{j-1}]}_{\xi} M_{i,t} + \varsigma_{i,t} + \Delta e_{i,t}.$$

Estimation based on:

$$E(g_{i,t}|P_{i,t} = P_{i,t-1} = 1, M_{i,t} = 0)$$
$$E(g_{i,t}|P_{i,t} = P_{i,t-1} = 1, M_{i,t} = 1).$$

Take into account: $\rho_{\varsigma\pi}, \rho_{\varsigma\mu}, \rho_{\xi\mu}, \rho_{\xi\pi}, \rho_{\xi\pi-1}$.
Need first and second moments of the twice truncated, multivariate (5) normal distribution.

Correcting for Selection III

1. Estimate probits: $X = \pi_{it}, \mu_{it}, \alpha z_{it}, \theta\kappa_{it}$.
2. Exclusion restrictions: *UI* at state level and unearned income.
3. Non-linear estimation of first and second moment:

$$h(\sigma_{\varsigma}, \sigma_e, \sigma_a, \rho_{\varsigma\pi}, \rho_{\varsigma\mu}, \rho_{\xi\mu}, \rho_{\xi\pi}, \rho_{\xi\pi-1}, X).$$

Correcting for Selection III

Example for identification:

$$E(g_{i,t}|P_{i,t} = P_{i,t-1} = 1, M_{i,t} = 0) = -\rho_{\varsigma\mu}\sigma_{\varsigma}\tilde{\lambda}^M_{i,t} + \rho_{\varsigma\pi}\sigma_{\varsigma}\lambda^P_{i,t}$$

- Assume people close to participation threshold, $\lambda^P_{i,t}$ small, have higher wage growth than those far away, $\lambda^P_{i,t}$ big.

  Estimate $\rho_{\varsigma\pi}$ negative.

- Assume people with high mobility, $\tilde{\lambda}^M_{i,t}$ big, have higher wage growth than those with little mobility, $\tilde{\lambda}^M_{i,t}$ small.

  Estimate $\rho_{\varsigma\mu}$ negative.

Results

	Whole sample (1)	Low education (2)	High education (3)	Neglect mobility (all) (4)
Standard deviations				
σ_ζ	0.103	0.095	0.106	0.152
	(0.012)	(0.022)	(0.017)	(0.009)
	[0%]	[1%]	[0%]	[0%]
σ_e	0.087	0.084	0.088	0.086
	(0.011)	0.035)	(0.016)	(0.005)
	[0%]	[0%]	[0%]	[0%]
σ_a	0.228	0.226	0.229	
	(0.011)	(0.019)	(0.015)	
	[0%]	[0%]	[0%]	

- 2 std deviations from match effects: Wages differ by 46%.
- Large effect on σ_ς compared to no mobility.
- Little difference by education.

Optimal Insurance

- Welfare implications of different risk types.
- How should government provide insurance?
- Temporary risk: Unemployment benefits.
- Permanent risk: Food Stamps and DI.

The Environment

- Estimated productivity process.
- Workers search on and off the job.
- Exogenous and endogenous separations.
- Self-insurance by asset accumulation.

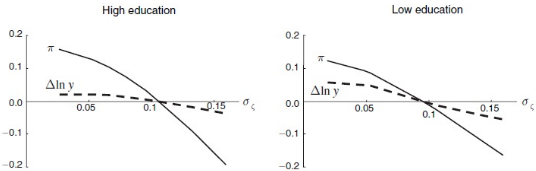


FIGURE 5. WELFARE COSTS AND OUTPUT EFFECTS OF VARYING σ_ζ

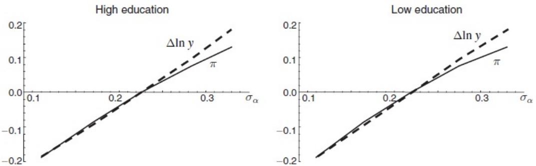


FIGURE 9. WELFARE COSTS AND OUTPUT EFFECTS OF VARYING FIRM HETEROGENEITY

- Wage risk decreases welfare (by more than output).
- Firm risk increases welfare (by less than output)!

Value of Governmental Insurance

Table 6—Welfare Effects of Government Programs

Scenario	High education: Willingness to pay percent ($\pi \times 100$)	Low education: Willingness to pay percent ($\pi \times 100$)
Unemployment insurance	0.19	0.24
Food stamps	0.25	0.30
Tax change	0.08	0.15

- Increasing welfare spending by 1%.
- Significant welfare gains from UB and Food Stamps.
- Food Stamps: Insurance against permanent risk.

Postel-Vinay and Robin (2002)

Basic framework

- On-the-job search model.
- Firms have heterogeneous productivities, p_j.
- Workers have heterogeneous productivities, ϵ_i.
- Continuum of competitive firms producing with constant returns to labor and technology.
- Hence, total output is the sum of all worker productivities times the firm productivity: $Y(p) = p \sum_{i=0}^{m} \epsilon_i$.
- Wages are endogenous: Firms post wages to maximize profits. The common alternative is a wage bargaining framework (DMP model).

The importance of wage determination

- Firms post wages to attract employed and unemployed workers.
- Key novelty: When an outside offer arrives, firms engage in Bertrand competition for the worker.
- Once a wage is negotiated, it cannot be changed until mutual consent.
- This implies that the same worker earns different wages at the same job depending on the history of outside offers.
- Here, tenure effects result from outside offers.
- Good jobs have high tenure effects and, hence: $Corr(\phi, \epsilon_{i,j,t}) \neq 0$.
- An alternative interpretation is that ϕ_j is not time invariant.

Workers

- Total mass of workers is M. Born and dye at rate μ.
- When born, draw a time invariant productivity ϵ from a distribution with CDF H.
- Unemployment inflow rate: $\mu M + \delta$.
- When unemployed, workers earn benefits proportional to her productivity: ϵb.

Matching and Wage Setting

- Unemployed sample job offers randomly at rate λ_0 and with rate λ_1 when employed. When matched, p randomly selected from CDF F.
- Firms set wages according to the following rules:
  - Wage offers may vary for different ϵ.
  - Any offer from an outside firm can be countered.
  - Firms make take-it-or-leave-it offers.
  - Renegotiation is only possible by mutual agreement.

Value Function: Unemployed

As workers dye at rate μ, their asset value discounts with $\rho + \mu$:

$$(\rho + \mu) V_0(\epsilon) = U(\epsilon b) + \lambda_0 \int \{V(\epsilon, \phi_0(\epsilon, p), p) - V_0(\epsilon)\} \, dF(p) \qquad (1)$$

- $U(\epsilon b)$ is the flow utility of unemployment benefits b.
- $\phi_0(\epsilon, p)$ is the wage contract a firm of type p will offer an unemployed worker.

Wage Contracts: Unemployed

Let $V(\epsilon, w, p)$ be the value function and employed worker with current wage w. Firms have all the bargaining power and make offers to the unemployed that make them indifferent:

$$V(\epsilon, \phi_0(\epsilon, p), p) = V_0(\epsilon)$$

- All firms make the unemployed indifferent to staying unemployed. Hence, the unemployed accept all offers.
- As a result, the reservation wage is independent of λ_0.

Value Function: Unemployed II

As workers are indifferent between any offer and being unemployed, we have $V(\epsilon, \phi_0(\epsilon, p), p) - V_0(\epsilon) = 0$.

$$V_0(\epsilon) = \frac{U(\epsilon b)}{r + \mu} \tag{2}$$

- The value of unemployment depends only on the worker's productivity ϵ. As it is increasing in ϵ, the value of employment is also increasing in ϵ, i.e., the wage offer is increasing in ϵ.

Wage Contracts: Employed

Before defining the value function of the employed, we have to think about outside offers. When an outside offer arrives, the most a firm can pay is the worker's full marginal product $w = \epsilon p$. A worker will move to a firm p' if that firm can promise her more value:

$$V(\epsilon, \phi(\epsilon, p, p'), p') > V(\epsilon, \epsilon p, p)$$

Otherwise, she will stay with the current firm. Importantly, an outside offer depends on current firm productivity and productivity of poaching firm: $\phi(\epsilon, p', p)$.

Wage Contracts: Employed II

- Define (for a given ϵ, p) a firm type $p' = q(\epsilon, w, p)$ such that the outside offer equals the current wage: $\phi(\epsilon, p, q(\epsilon, w, p)) = w$.
- The most this firm can offer to the worker is her marginal product $p'\epsilon$. Hence, for the worker
  - nothing changes if $p' < q(\epsilon, w, p)$.
  - The wage rises to $w = \epsilon p'$ if $p \geq p' \geq q(\epsilon, w, p)$.
  - The worker moves and gets wage $\phi(\epsilon, p, p')$ if $p' > p$. The outside firm will make her indifferent between moving and staying: $V(\epsilon, \phi(\epsilon, p, p'), p') = V(\epsilon, \epsilon p, p)$.

Value Function: Employed

The state of an employed is her productivity, the firm's productivity, and the current wage:

$$
\begin{aligned}
(\rho+\mu)V(\epsilon,w,p) = U(w) &+ \lambda_1 \int_{p}^{\infty} \{V(\epsilon,\epsilon p,p) - V(\epsilon,w,p)\}\, dF(p') \\
&+ \lambda_1 \int_{q(\epsilon,w,p)}^{p} \{V(\epsilon,\epsilon q(\epsilon,w,p),p) - V(\epsilon,w,p)\}\, dF(p') \\
&+ \delta\left[V_0(\epsilon) - V(\epsilon,w,p)\right]. \qquad (3)
\end{aligned}
$$

Value Function: Employed II

Evaluate the function at $w = \epsilon p$. In that case:

$$V(\epsilon, \epsilon p, p) - V(\epsilon, w, p) = 0 \tag{4}$$

$$\int_{q(\epsilon,w,p)}^{p} dF(p') = 0 \tag{5}$$

and we have

$$V(\epsilon, \epsilon p, p) = \frac{U(\epsilon p) + \delta V_0(\epsilon)}{\rho + \mu + \delta} \tag{6}$$

$$V'(\epsilon, \epsilon p, p) = \epsilon \frac{U'(\epsilon p)}{\rho + \mu + \delta}. \tag{7}$$

Value Function: Employed III

Using integration by parts we have:

$$
\begin{aligned}
(\rho+\mu+\delta)V(\epsilon,w,p) = U(w) + \delta V_0(\epsilon) \\
+ \lambda_1(1-F(p))[V(\epsilon,\epsilon p,p) - V(\epsilon,w,p)] + \lambda_1\Big[F(p)[V(\epsilon,\epsilon p,p) \\
- V(\epsilon,w,p)] - \int_{q(\epsilon,w,p)}^{p} V'(\epsilon,\epsilon q(\epsilon,w,p),p)F(p')dp'\Big] \qquad (8) \\
= U(w) + \delta V_0(\epsilon) + \lambda_1\Big[\int_{q(\epsilon,w,p)}^{p} V'(\epsilon,\epsilon q(\epsilon,w,p),p)dp' \\
- \int_{q(\epsilon,w,p)}^{p} V'(\epsilon,\epsilon q(\epsilon,w,p),p)F(p')dp'\Big] \qquad (9)
\end{aligned}
$$

Value Function: Employed IV

This finally simplifies to

$$(\rho+\mu+\delta)V(\epsilon,w,p) = U(w)+\delta V_0(\epsilon) + \frac{\lambda_1\epsilon}{\rho+\mu+\delta}\int_{q(\epsilon,w,p)}^{p} U'(\epsilon p')(1-F(p'))dp' \quad (10)$$

The value depends on:

- the flow value of the current wage, w.
- the value of outside options between the reservation productivity and p. These outside offers increase workers' wages.

Reservation Productivity

To derive the reservation productivity $q(\epsilon, w, p)$, assume current productivity is equal to the reservation productivity $p = p' = q(\epsilon, w, p)$:

$$V(\epsilon, w, p) = V(\epsilon, \epsilon q(\epsilon, w, p), q(\epsilon, w, p)) = \frac{U(\epsilon q(\epsilon, w, p)) + \delta V_0(\epsilon)}{\rho + \mu + \delta}$$

$$(\rho + \mu + \delta) V(\epsilon, w, p) = U(w) + \delta V_0(\epsilon) + \frac{\lambda_1 \epsilon}{\rho + \mu + \delta} \int_{q(\epsilon,w,p)}^{p} U'(\epsilon p')(1 - F(p'))dp'$$

$$U(w) = U(\epsilon q(\epsilon, w, p)) - \frac{\lambda_1 \epsilon}{\rho + \mu + \delta} \int_{q(\epsilon,w,p)}^{p} U'(\epsilon p')(1 - F(p'))dp' \quad (11)$$

- The equation gives us an implicit solution for the reservation productivity.
- As intuition would suggest, $q(\epsilon, w, p) = p$.

Solution to the Wage Contract

Now consider a wage offer $w = \phi(\epsilon, p, p')$ for $p' \geq p$. We know this firm will pay the worker its reservation wage, i.e., $q(\epsilon, \phi(\epsilon, p, p'), p') = p$:

$$U(\phi(\epsilon, p, p')) = U(\epsilon p) - \frac{\lambda_1 \epsilon}{\rho + \mu + \delta} \int_p^{p'} U'(\epsilon p')(1 - F(p'))dp' \qquad (12)$$

- This is a closed-form solution for the wage contract $\phi(\epsilon, p, p')$. This makes the model very fast to solve!
- Workers accept lower wages when going to more productive firms. They get compensated by future expected wage growth.
- Workers may even take a wage cut.

Solution to the Wage Contract II

For the unemployed, their previous productivity was b. Hence,

$$U(\phi(\epsilon, b, p')) = U(\epsilon b) - \frac{\lambda_1 \epsilon}{\rho + \mu + \delta} \int_b^{p'} U'(\epsilon p')(1 - F(p'))dp' \qquad (13)$$

- Key to this is that workers receive ϵb in unemployment, i.e., more productive workers have higher income during unemployment.
- The starting wage after unemployment is decreasing in p'.

Steady-State

Let $l(p)$ be the density of workers at type p firms with CDF $L(p)$ and define $\kappa_1 = \lambda_1/(\delta + \mu)$. Then steady state implies:

$$u = \frac{\delta + \mu}{\delta + \mu + \lambda_0}$$

Distribution of firm types across workers:

$$L(p) = \frac{F(p)}{1 + \kappa_1(1 - F(p))}$$

The Data Set

- Matched employer-employee data from the French private sector from 1996-1998.
- Firms with more than 5 employees from the district Ile-de-France.
- Seven categories of workers based on tasks.

Descriptive Analysis

Occupation	Number of indiv. trajectories	Percentage with no recorded mobility (%)	Percentage whose first recorded mobility is from job... ...to-job (%)	...to-out of sample (%)	Sample mean unemployment spell duration	Sample mean employment spell duration
Executives, managers, and engineers	22,757	46.2	23.4	30.4	0.96 yrs	2.09 yrs
Supervisors, administrative, and sales	14,977	48.1	19.3	32.5	1.16 yrs	2.11 yrs
Technical supervisors and technicians	7,448	55.5	16.0	28.6	1.07 yrs	2.28 yrs
Administrative support	14,903	54.3	8.2	37.5	1.30 yrs	2.23 yrs
Skilled manual workers	12,557	55.9	5.2	38.9	1.16 yrs	2.28 yrs
Sales and service workers	5,926	45.1	5.5	49.4	1.28 yrs	2.06 yrs
Unskilled manual workers	4,416	42.5	7.0	50.5	1.29 yrs	1.98 yrs

Estimation Method

- Identifying assumption:
  - Wage observations independent draws from the wage distribution.
  - Mean earning utility $y(p) = E[U(w)|p]$ is increasing function in p. I.e., I can rank firms by mean wages.
  - No sampling errors in within-firm mean earning utilities y_j.
  - CRRA preferences give $ln(\phi(\epsilon, p, p')) = ln(\epsilon) + ln(\phi(1, p, p'))$.
  - Restrict to firms with more than 5 employees.
- Compute transition probabilities by maximizing the likelihood by type.
- Get an estimate of p_j given y_j and ρ.
- Estimate the distribution of $ln(\phi(1, q_i, p_i))$.

Variance Decomposition

- Decompose:
  $Var\Big(ln(w)\Big) = Var\Big(ln(\epsilon)\Big) + Var\Big(E(ln(w)|p)\Big) + E\Big(Var\Big(ln(w)|p\Big)\Big).$
- Individual effect, between firm effect, within firm effect.

Result

Occupation	Nobs.	Mean log wage: $E(\ln w)$	Total log-wage variance/coeff. var.: $V(\ln w)$	CV	Case $U(w) =$	Firm effect: $VE(\ln w \mid p)$ Value	Firm effect: % of $V(\ln w)$	Search friction effect: $EV(\ln w \mid p) - V \ln \varepsilon$ Value	Search friction effect: % of $V(\ln w)$	Person effect: $V \ln \varepsilon$ Value	Person effect: % of $V(\ln w)$
Executives, manager, and engineers	555,230	4.81	0.180	0.088	$\ln w$	0.035	19.3	0.082	45.5	0.063	35.2
					w	0.035	19.4	0.070	38.7	0.076	41.9
Supervisors, administrative and sales	447,974	4.28	0.125	0.083	$\ln w$	0.034	27.5	0.065	52.1	0.025	20.3
					w	0.034	27.9	0.069	55.1	0.022	17.8
Technical supervisors and technicians	209,078	4.31	0.077	0.064	$\ln w$	0.025	32.4	0.044	57.6	0.008	10.0
					w	0.025	32.8	0.047	60.6	0.005	6.6
Administrative support	440,045	4.00	0.082	0.072	$\ln w$	0.029	35.7	0.043	52.2	0.010	12.1
					w	0.028	34.6	0.045	55.7	0.008	9.7
Skilled manual workers	372,430	4.05	0.069	0.065	$\ln w$	0.029	42.9	0.039	57.1	0	0
					w	0.028	41.5	0.040	58.5	0	0
Sales and service workers	174,704	3.74	0.050	0.060	$\ln w$	0.020	40.8	0.029	58.7	0.0002	0.4
					w	0.019	37.1	0.029	57.9	0.0025	5.0
Unskilled manual workers	167,580	3.77	0.057	0.063	$\ln w$	0.027	48.3	0.029	51.7	0	0
					w	0.023	40.8	0.033	59.2	0	0

- Search frictions: About 50%.
- Firm effects: 50% for low skilled and 20% for high skilled.
- Person effect only important for high skilled.

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Heckit Model

Assume we are interested in education $educ_i$ on wages of females y_i

$$y_i = x_i'\beta + \epsilon_i \quad \epsilon_i \sim N(0, \sigma_\epsilon^2) \quad educ_i \in x_i$$

Wages are only observed when females participate. Define the latent variable

$$z_i^* = w_i'\gamma + u_i \;\; z_i = 1 \;\; if \;\; z_i^* > 0 \quad u_i \sim N(0, \sigma_u^2) \quad educ_i \in w_i$$

The problem intuitively:

Part of the error, ϵ_i is motivation.
Motivation is also part of u_i.
Even when motivation is random in the population, the subsample of low educated workers working is more motivated than than those with high education.

Heckit Model II

$$
\begin{aligned}
&y_i = x_i'\beta + \epsilon_i \;\; \textit{observed if} \;\; z_i = 1 \\
&z_i^* = w_i'\gamma + u_i \;\; z_i = 1 \;\; \textit{if} \;\; z_i^* > 0 \\
&Prob(z_i = 1|w_i) = \mathbf{\Phi}(w_i'\gamma) \\
&(u_i, \epsilon_i) \sim \;\; \textit{bivariate normal} \;\; [0, 0, 1, \sigma_\epsilon^2, \rho].
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[y_i|z_i^* = 1] &= \mathbb{E}[y_i|u_i > -w_i'\gamma] \\
&= x_i'\beta + \mathbb{E}[\epsilon_i|u_i > -w_i'\gamma]
\end{aligned}
$$

Heckit Model III

Moments of the incidentally (upper) truncated bivariate normal distribution:

$$
\begin{aligned}
\mathbb{E}[y|z > a] &= \mu_y + \rho\sigma_y\lambda(\alpha_z) \\
Var[y|z > a] &= \sigma_y^2[1 - \rho^2\delta(\alpha_z)] \\
\alpha_z &= \frac{a - \mu_z}{\sigma_z} \\
\lambda(\alpha_z) &= \frac{\phi(\alpha_z)}{1 - \Phi(\alpha_z)} \\
\delta(\alpha_z) &= \lambda(\alpha_z)[\lambda(\alpha_z) - \alpha_z]
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[y_i|z_i^* = 1] &== x_i'\beta + \mathbb{E}[\epsilon_i|u_i > -w_i'\gamma] \\
&= x_i'\beta + \rho\sigma_\epsilon\lambda(-w_i'\gamma) \\
&= x_i'\beta + \beta_\lambda\lambda(-w_i'\gamma)
\end{aligned}
$$

Back